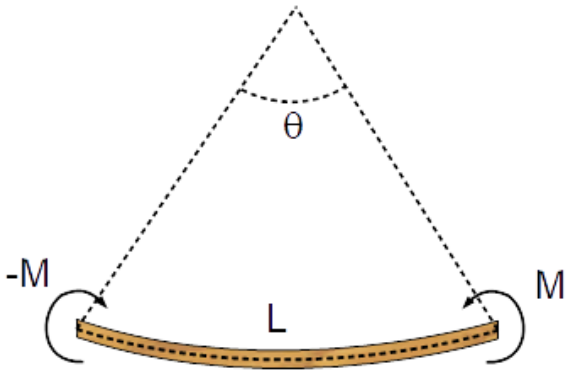
Ruler Bending About its Own Centroid

If you bend a beam with a pure moment, you bend it into an arc of a circle. The angle θ described is a measure of how much the ruler bends.



$\theta_{flat} > \theta_{tall}$ for the the same moment. What moments do we need to get the same deflections?

Let's assume the formula give above: the moment needed to bend a ruler into a given arc θ is

$$M = \theta\left(\frac{EI}{L}\right)$$

E is a material property, but the material of my ruler hasn't changed if I rotate it 90 degrees. The length hasn't changed either. If I set the θ's equal to each other, the only thing left is I.

If we wanted to bend the flat ruler and the tall ruler to similar arcs (same θ's), how much bigger does M have to get?

$\theta_{tall} = \theta_{flat}$

$M_yL/EI_y = \theta_{tall} = \theta_{flat} = M_xL/EI_x$

Cancel L and E from both sides:

$M_y/I_y = M_x/I_x$

For bending the ruler flat-wise (about the z-axis), we need to find I_x.

The cross-section of the ruler is about one inch by a quarter inch. With the z axis through the ruler, the cross-section is a normal x-y axes system. The cross-section is a ruler with base = 1 inch and height = 0.25 inches. For bending the ruler flat-wise,

$I_x = 1/12 * 1* 0.25^3 = 0.001302\ in^4$

For bending the beam tall-wise (about the y-axis), we need to find I_y: the cross-section of bending this way is 0.25 inches as a base and 1 inch as a height.

$I_y = 1/12 * 1^3* 0.25= 0.020833\ in^4$

$M_y/0.020833 = M_x/0.001302$

$M_y = 16\ M_x$

For the ruler above, it takes 16 times more moment to bend the ruler into the same arc. If you consider a ruler that is 1.25 inches wide and 0.2 inches thick, just slightly wider and thinner, this factor increases to 39 times more moment. This comes directly from the moments of inertia.

As we move the area of the cross-section farther from the neutral axis, the moment of inertia increases. As I increases, $\theta = ML/EI$ decreases. It's harder to bend a ruler tall-wise.

Does the beam bend more or less if the I increases? I is the resistance to bending. If I increases, the beam is more resistant to bending so it bends less.