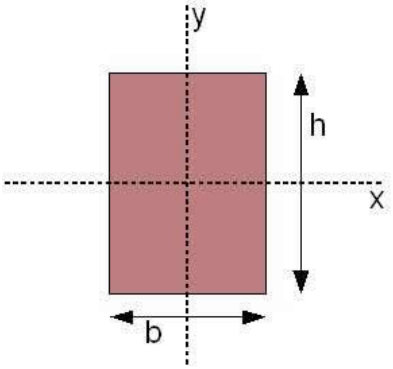
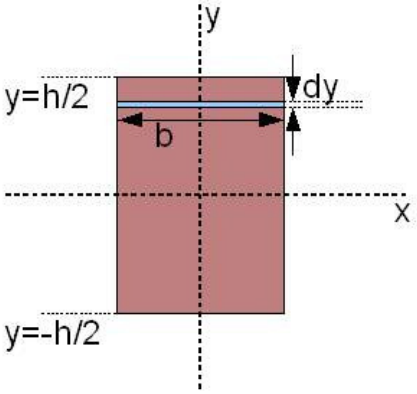
Moment of Inertia Calculation by Integration
Rectangular Beam Bending about Centroidal Axes

The area moment of inertia is determined by the cross-section of the beam. The cross-section of a ruler is essentially a rectangle. Find the moment of inertia of the rectangle about its centroid:



First, find the differential area you want to use.



The area of that differential stripe (base * height) = dA = b*dy

Plug this into the definition of the moment of inertia:

$$I_x = \int_{-h/2}^{h/2} y^2 b\,dy$$

Evaluate the integral:

$$I_x = \frac{1}{3} y^3 b \Big|_{-h/2}^{h/2}$$

$$I_x = \frac{1}{12} b h^3$$

This is the moment of inertia for a rectangle about its centroid. Memorize this.

If you calculate the moment of inertia about the y axis, the differential area is a vertical stripe. So dA = h * dx. Finishing the integral gives

$$I_y = \frac{1}{12} b^3 h$$

So we learn that a rectangle area moment of inertia about its centroid is the area of the rectangle over twelve times the dimension away from the axis of bending squared.

Remember to cube the distance perpendicular to the axis you're calculating the moment of inertia about.

https://youtu.be/BvEu0fpLfdc