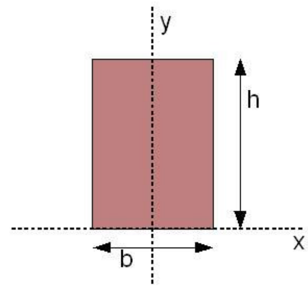
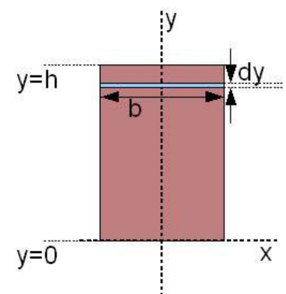
Moment of Inertia Calculation by Integration

Rectangular Beam Bending About Baseline

Find the moment of inertia of the rectangle about its baseline:



First find the differential area you want to use for the integration:



The area of the differential element dA = b * dy as it was when we were bending about the centroid.

The definition of the moment of inertia:

$$I_x = \int_0^h y^2 b\, dy$$

Evaluate the integral:

$$I_x = \frac{1}{3} y^3 b \Big|_0^h$$

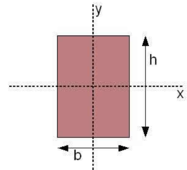
$$I_x = \frac{1}{3} b h^3$$

This is the moment of inertia about the rectangle baseline. Notice that you get a different answer depending on which x axis you take. The thing to remember here is that, even though the centroid of a shape is independent of its axes, the area moment of inertia is definitely not.

https://youtu.be/nD1ZMlrgJ_k

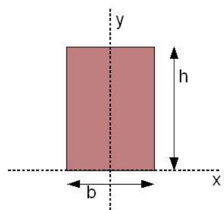
Why is the moment of inertia about a rectangle's baseline bigger than about its centroid?

If you bend a beam the beam to the right about the middle (assume we're applying a moment in x), the material at the top has to compress while the material at the bottom has to stretch.



The farther off the axis your material is, the more that compression/extension must occur -- and it increases **geometrically** not linearly. The moment of inertia is proportional to the distance off the bending-axis *cubed*. The material of the beam near the axis doesn't have to do much stretching or compressing.

Now, if you bend the beam about the bottom, this would be like trying to lay a beam on a table – an undeformable table – and trying to bend the beam lying on it. The material at the top has to compress a whole bunch. You've also got half the material near the x axis where the compression/extension is smaller.



So the resistance to bending will be smallest when the neutral axis passes through the centroid of your shape.