Parallel Axis Theorem

We don't always want to have to integrate. The parallel axis theorem lets us relate the moment of inertia about any axis x to the moment of inertia of the area about an x' axis which is parallel to the x axis and goes through the centroid.

$$I_x = I_{x'} + Ad^2$$

where

- I_x is the moment of inertia about your x axis (any axis parallel to the x axis passing through the centroid),
- $I_{x'}$ is the moment of inertia about the x'-axis which is parallel to the x-axis and passes through the centroid,
- A is the area of your shape,
- and d is the perpendicular distance between the x axis and the x' axis.

So, for our rectangle above,

- I_x would be the moment of inertia about the baseline.

$$I_x = \frac{1}{3}bh^3$$

- $I_{x'}$ would be the moment of inertia about the centroidal x-axis.

$$I_{x'} = \frac{1}{12}bh^3$$

- A = b*h
- d = h/2

So

$$\frac{1}{3}bh^3 = \frac{1}{12}bh^3 + bh\left(\frac{h}{2}\right)^2$$

The formula is the same for the moment of inertia about the y axis:

$$I_y = I_{y'} + Ad^2$$

Note that the smallest moment of inertia for an object is always about its centroidal axes. Since I_y = the centroid + Ad^2 and since A and d^2 are always positive, the centroidal moment of inertia must be smallest.