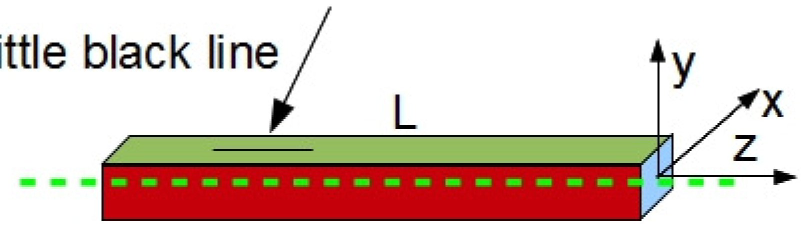
Neutral Axis -- Optional

Look at the little black line in the green face on the top of the beam.

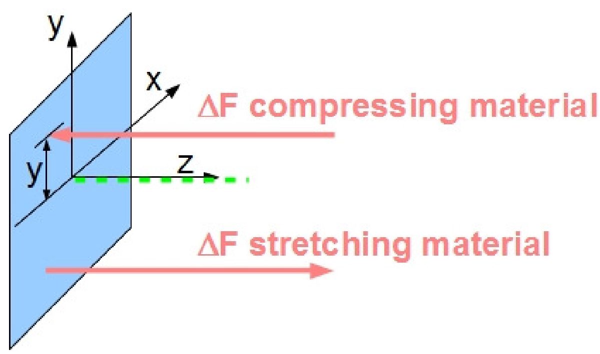


If you bend the beam about the x-axis, that line is going to get shorter. The material on the top is in compression. A similar line on the bottom of the beam would have to stretch. So the material on the bottom is in tension. Somewhere in the middle of the beam there has to be a line which wouldn't change its length at all: that's the line we call the neutral axis. It goes through the centroids of the cross-section.

The neutral axis is the line where there is no change in length along the z axis.

Now let's consider the blue face.

The top of the blue rectangle (material with positive y coordinates) is in compression. The bottom is in tension. So, inside the beam, there have to be internal forces making these tensions and compressions:



In solids you'll actually write the sum of the forces and the sum of the moments to show how these bits of forces contribute. For now, just think of what would happen if you tried to integrate these bits of force over the face of the blue side: ΔF has to be a function of y since it's biggest at the top, zero in the middle, and biggest in the opposite direction at the bottom.

Since ΔF is already proportional to y, adding it up all the way through means that the total force is proportional to y^2. And the total moment would have yet another factor of y and be proportional to y^3.

The total force ends up being proportional to the first moment of the area:

$$Q_x = \int y dA$$

The total moment on this face is proportional to the second moment of the area:

$$I_x = \int y^2 dA$$

You can see here that the more material you have farther off the neutral axis, the bigger I will get.