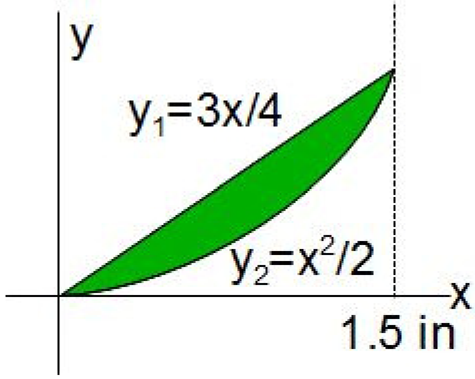
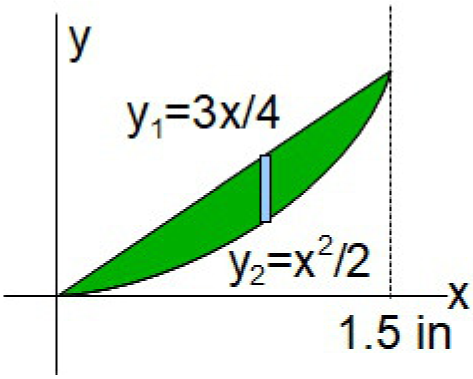
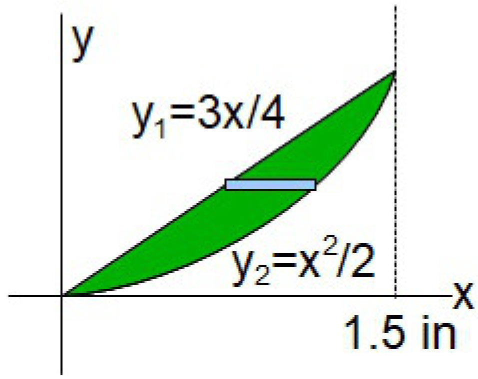
Moment of Inertia Calculation
Area Between Two Curves

Find the moment of inertia for the green shape about the x and y axes. The units shown are in inches; the drawing is not to scale.



Solution:





The vertical dA is used to find I_y. The height of this rectangle is $y_1 - y_2$ or $3x/4 - x^2/2$. The width is dx.

$$I_y = \int x^2 dA = \int_0^{1.5} x^2 \left(\frac{3x}{4} - \frac{x^2}{2} \right) dx$$

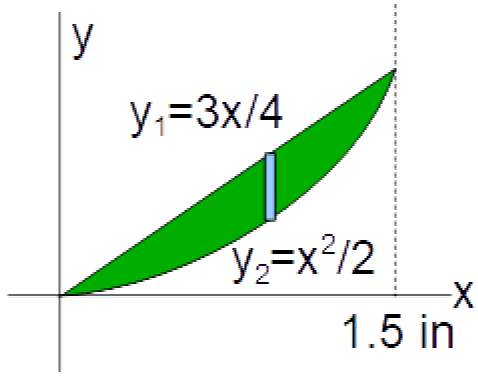
$$I_y = 0.190 \text{ in}^4$$

The horizontal dA can be used to find I_x. The width of this rectangle is $x_2 - x_1$ or sqrt(2y) – 4y/3. The height is dy.

$$I_x = \int y^2 dA = \int_0^{1.125} y^2 \left(\sqrt{2y} - \frac{4y}{3} \right) dy$$

$$I_x = 0.0763 \text{ in}^4$$

Note: we had to solve each of the functions for x in terms of y to be able to use this integral. Sometimes this is difficult. It would be great to be able to use the vertical element for finding I_x. To do that, integrate dI_x. That is, find the moment of inertia for the blue vertical rectangle about the x axis.



The blue rectangle is just a rectangle. So we can use the parallel axis theorem plus 1/12 bh^3 to express the total moment of inertia for the rectangle about the x axis. The parallel axis theorem says that the moment of inertia is the centroidal value + Ad^2 where d is the distance from the axis to the centroid (measured perpendicularly from the axis.)

$$I_x \neq \int y^2 dA$$

$$I_x = \int dI_x = \int \left[\frac{1}{12} base \cdot height^3 + Area \cdot d^2 \right] \text{ since } dA \text{ is a rectangle}$$

$$d = \frac{x^2}{2} + \frac{height}{2} \text{ (go from the x axis to the bottom curve \& halfway up)}$$

$$I_x = \int_0^{1.5} \left[\frac{1}{12} dx \cdot \left(\frac{3x}{4} - \frac{x^2}{2} \right)^3 + dx \cdot \left(\frac{3x}{4} - \frac{x^2}{2} \right) \cdot \left(\frac{x^2}{2} + \frac{1}{2} \left(\frac{3x}{4} - \frac{x^2}{2} \right) \right)^2 \right]$$

$$I_x = 0.0763 \text{ in}^4$$

Warning: sometimes students get creative here in trying to figure out how to use the integral y^2dA. Please avoid this.